SCIPER: ____________________

First name: ________________________ Family name: ________________________

EXAM SMART GRID TECHNOLOGIES 2023 (Total Duration: 3 hours)
With Solutions

PROBLEM 1 (WEIGHT: 20 %)

QUESTION 1

a.) A 50 Hz signal (1 p.u. amplitude) with an interfering tone at 600 Hz (0.2 p.u. amplitude) is sampled at 1 kHz. Sketch the expected spectrum in Figure 1 and briefly explain any phenomena present.

Solution. The 600 Hz tone will appear at $F_s - 600 = 400$ Hz because the frequency of the interfering tone exceeds the Nyquist limit of $F_s/2$. The negative image of the spectral copy centered around F_s will overlap with the positive image of the spectral copy centered around 0 Hz. This is an example of aliasing.

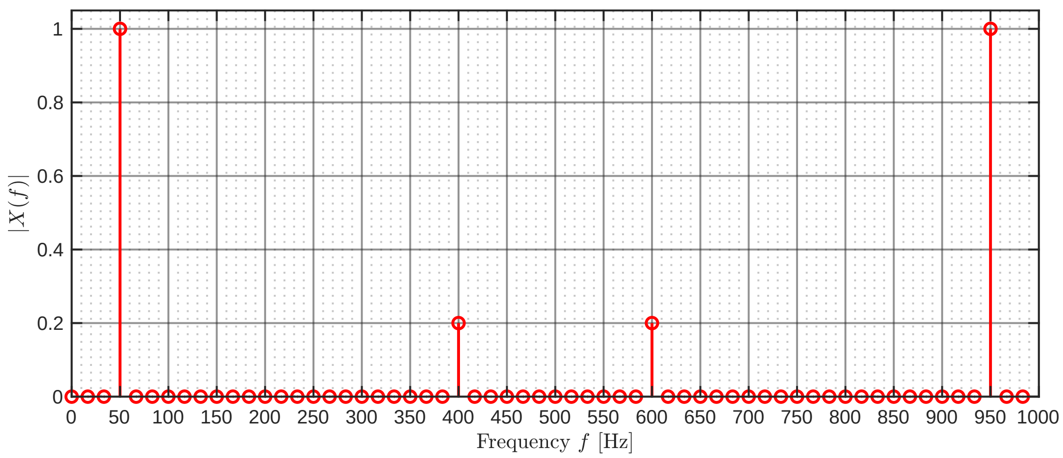

Figure 1: Frequency spectrum

b.) Considering the discrete spectrum (ignoring any post-processing techniques, like IpDFT), provide an example of a scenario where

(i) the Rectangular window outperforms the Hanning window:

Solution. The Rectangular window will allow the observer to better distinguish multiple tones that are close to each other in the frequency spectrum due to the narrow main lobe of the window.

(ii) the Hanning window outperforms the Rectangular window:

Solution. Thanks to the lower sidebands, the Hanning window will better attenuate spectral leakage caused by incoherent sampling of any frequency tones.

c.) Given a 52 Hz signal, choose a window length and windowing function for analysis with DFT. Justify your choice and describe the effect on the spectrum.

Solution. There are several possible options that could yield acceptable results in this scenario:

1. The window could be selected to contain an integer multiple of cycles of the fundamental tone ($K * 1/f_0$) to ensure coherent sampling. Rectangular or Hanning window would yield equivalent results in this case.

2. If a standard 60 ms window is used, there will be spectral leakage due to the incoherent sampling of the signal. This could be minimized using a special windowing function, i.e., the Hann window.

3. A very large window could be selected to reduce the frequency resolution, ensuring that the true frequency is close to a frequency bin. However, unless the signal is coherently sampled there will still be spectral leakage which can be attenuated by the Hanning windowing function. Note that a large window will increase the latency and computational complexity of the algorithm in a PMU and the stationary waveform assumption is less likely to be valid for long observation windows.

d.) A 52 Hz signal is now analyzed with a 60 ms window using IpDFT. What is the sign of the fractional correction coefficient δ? Explain your reasoning.

Solution. $\Delta f = 16.67$ Hz and therefore the maximum bin is located at $k = 3$ (50 Hz) and the next largest bin is located at $k = 4$ (66.66 Hz). The correction coefficient will then be positive.

**With the information provided, we must assume that an appropriate windowing function is used such that the spectral leakage from the negative image is not enough to significantly corrupt the bins and change the sign. Furthermore, since the true frequency is further from DC than in the case of Q1 (Exam 2021), even using the rectangular window will likely yield the correct sign.

Question 2

A sinusoid at $f_0 = 50$ Hz is analyzed by two PMUs. Both PMUs receive a GPS signal that triggers the reporting of the phasor estimate at a rate of $F_r = 50$ fps (frames per second). Both PMUs process a window of $N = 600$ samples for IpDFT calculations.

a.) Based on the provided reporting rate and the UTC absolute time, sketch the subPPS square wave with the rising edge corresponding to the reporting trigger in Figure 2 .

b.) PMU 1 has a base clock that is perfectly synchronized to UTC absolute time and a sampling frequency of $F_{s,1} = 10$ kHz. What is the sampling period $T_{s,1}$ and window length $T_{w,1}$ of PMU 1? Draw 5 consecutive windows in Figure 2 (starting with the first fully visible window).

Solution. $T_{s,1} = 100$ μs, $T_{w,1} = 60$ ms.

c.) The base clock of PMU 2 has an offset frequency, such that its true sampling frequency is $F_{s,2} = 9.23$ kHz. What is the sampling period $T_{s,2}$ and window length $T_{w,2}$ of PMU 2? Draw 5 consecutive windows in Figure 2 (starting with the first fully visible window).

Solution. $T_{s,2} = 108.34$ μs, $T_{w,2} = N/F_{s,2} = 65$ ms. Since $T_w = N/F_s$, if $F_{s,2} < F_{s,1}$, then $T_{w,2} > T_{w,1}$. In figure 2, since the reporting trigger is shared by both PMUs, the ends of both reported windows should correspond to the rising edge of the subPPS.

d.) Qualitatively, how does the frequency estimate $\hat{f}_0$ of PMU 1 compare to that of PMU 2?

Solution. PMU 2 has a lower sampling frequency (i.e., larger sampling time), so $N = 600$ samples corresponds to a longer window in reference time which PMU 2 will interpret as a "compressed signal", leading to the estimation of a higher fundemantal frequency than the reference. The phase estimated by PMU 2 is also offset because the observed window starts at a different phase than in PMU 1. While it is true that the observed window is no longer coherently sampled, the effects of spectral leakage are likely handled by a special windowing function. However, even with a windowing function, the estimated frequency would not be correct due to the issues described above.

e.) How can we correct for the error in PMU 2's frequency estimation?

Solution. PMU 2 assumes that it has the correct 10 kHz sampling frequency and a window length of 60 ms when, in reality, both are off. While we likely cannot correct the true sampling frequency of the device, we can account for this error in the IpDFT calculations that assume a frequency resolution of $\Delta f = 1/T_w$. By adjusting this value to reflect the true window length, the frequency estimation will be correct.

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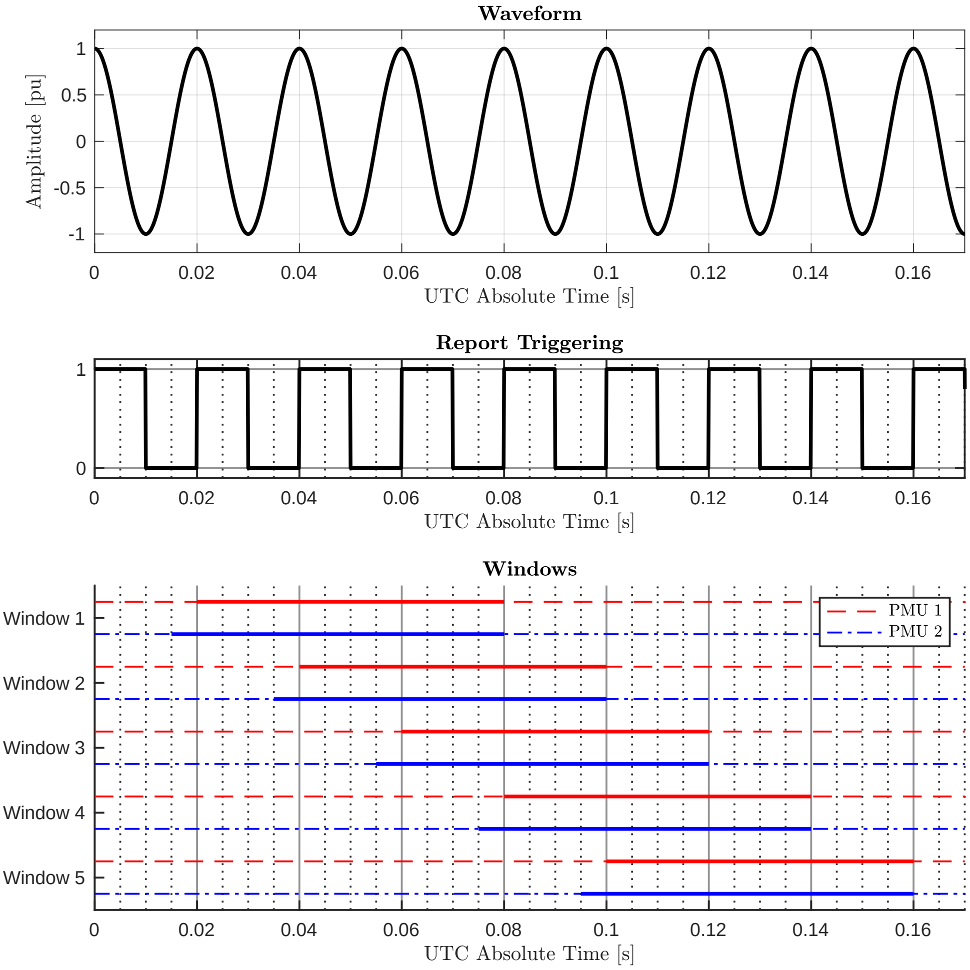

Figure 2: Comparison of reporting windows for PMUs 1 and 2 .

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EXAM SMART GRIDS TECHNOLOGIES 2023 (Total Duration: 3 hours)
With Solutions

PROBLEM 3 (WEIGHT: 30 %)

QUESTION 1 (PER-UNIT AND ADMITTANCE MATRIX CALCULUS)

Transformers in Per-Unit System

Consider the power system shown in Fig. 1, which consists of a synchronous machine (SM), two transformers (TF1 and TF2), a transmission line (TL), and a load (L). The transformers divide the system into three subsystems. To solve the circuit in per-unit, Student A and Student B have to decide on base quantities for each subsystem as indicated in Fig. 1.

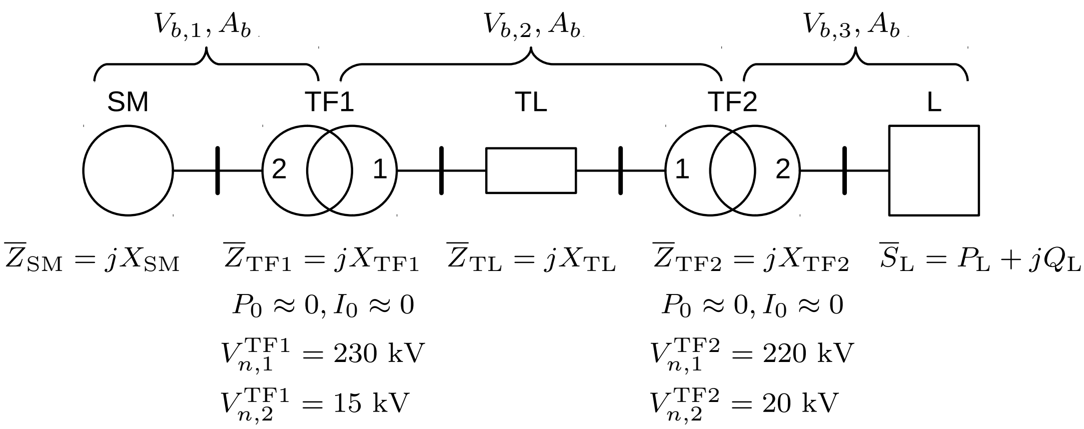


Figure 1: Power system with two transformers.

For such nominal voltages of the transformers TF1 and TF2 (indicated in Fig. 1), Student A claims that it is possible to set base voltages as $V_{b,1} = V_{b,1}^{\star}$, $V_{b,2} = V_{b,2}^{\star}$ and $V_{b,3} = V_{b,3}^{\star}$ such that the corresponding per-unit equivalent circuits of transformers do not contain any shunts (i.e., they can be represented by *simple models*, only by serial impedances $\overline{z}_{\mathrm{TF1}} = jx_{\mathrm{TF1}}$ and $\overline{z}_{\mathrm{TF2}} = jx_{\mathrm{TF2}}$), and therefore, proposes the complete per-unit equivalent circuit of the power system as shown in Fig. 2:

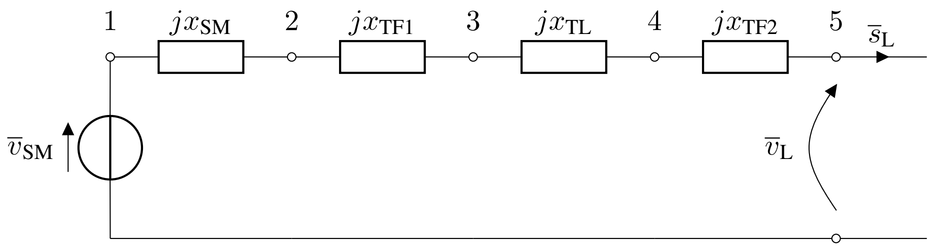


Figure 2: Per-unit equivalent circuit.

However, Student B opposes and states that the equivalent circuit is incomplete and that at least one transformer's model has to contain non-zero shunt elements for any choice of base voltages.

Q1.1 Whose reasoning is correct? Tick one box (no penalty, but requires an explanation):

☑ Student A's reasoning is correct.
Explain why and propose values for $V^{\star}_{b,1}$, $V^{\star}_{b,2}$ and $V^{\star}_{b,3}$.

☐ Student B's reasoning is correct.
Explain why and propose any possible complete per-unit equivalent circuit according to Student B.

Answer:

Solution.

Student A's reasoning is correct. To represent transformers with simple models that contain no shunts, the necessary conditions are

$$\frac{V_{b,1}}{V_{b,2}} = \frac{V^{\mathrm{TF1}}_{n,2}}{V^{\mathrm{TF1}}_{n,1}} \quad \text{and} \quad \frac{V_{b,2}}{V_{b,3}} = \frac{V^{\mathrm{TF2}}_{n,1}}{V^{\mathrm{TF2}}_{n,2}}, \tag{1}$$

which is possible to satisfy. One solution is $V^{\star}_{b,1} = 15$ kV, $V^{\star}_{b,2} = 230$ kV and $V^{\star}_{b,3} = \frac{230}{11} \approx 20.91$ kV. Every triplet $(kV^{\star}_{b,1}, kV^{\star}_{b,2}, kV^{\star}_{b,3})$ also satisfies (1), for any $k \in \mathbb{R}^{+}$.

Q1.2 Derive the admittance matrix corresponding to the per-unit equivalent circuit in Fig. 2 (respect the given numbering of buses).

Solution. The admittance matrix is as follows:

$$\overline{\mathbf{Y}} = \begin{bmatrix} \overline{y}_{\mathrm{SM}} & -\overline{y}_{\mathrm{SM}} & 0 & 0 & 0 \\ -\overline{y}_{\mathrm{SM}} & \overline{y}_{\mathrm{SM}} + \overline{y}_{\mathrm{TF1}} & -\overline{y}_{\mathrm{TF1}} & 0 & 0 \\ 0 & -\overline{y}_{\mathrm{TF1}} & \overline{y}_{\mathrm{TF1}} + \overline{y}_{\mathrm{TL}} & -\overline{y}_{\mathrm{TL}} & 0 \\ 0 & 0 & -\overline{y}_{\mathrm{TL}} & \overline{y}_{\mathrm{TL}} + \overline{y}_{\mathrm{TF2}} & -\overline{y}_{\mathrm{TF2}} \\ 0 & 0 & 0 & -\overline{y}_{\mathrm{TF2}} & \overline{y}_{\mathrm{TF2}} \end{bmatrix}, \tag{2}$$

where $\overline{y}_i = \dfrac{1}{jx_i}, i \in \{\mathrm{SM}, \mathrm{TF1}, \mathrm{TL}, \mathrm{TF2}\}$.

Q1.3 *Reconstruction of the Admittance Matrix*

Consider a generic electrical grid of N buses, $N \geqslant 2$. The electrical grid is described by its nodal admittance matrix $\overline{\mathbf{Y}}$ expressed as

$$\overline{\mathbf{Y}} = \mathbf{A}_{\mathfrak{B}}^{\top} \overline{\mathbf{Y}}_{\mathcal{L}} \mathbf{A}_{\mathfrak{B}} + \overline{\mathbf{Y}}_{\mathcal{T}}, \tag{3}$$

where $\mathbf{A}_{\mathfrak{B}}$ denotes the branch incidence matrix, $\overline{\mathbf{Y}}_{\mathcal{L}}$ the primitive branch admittance matrix, and $\overline{\mathbf{Y}}_{\mathcal{T}}$ the primitive shunt admittance matrix.

Is it possible to fully reconstruct the admittance matrix $\overline{\mathbf{Y}} \in \mathbb{C}^{N \times N}$ describing the grid (i.e., obtain all the elements) by knowing *only* its diagonal elements $\overline{Y}_{11}, \ldots, \overline{Y}_{NN}$ and

(a) $\overline{\mathbf{Y}}_{\mathcal{T}}$?

(b) $\overline{\mathbf{Y}}_{\mathcal{T}}$ and $\mathbf{A}_{\mathfrak{B}} \in \mathbb{R}^{L \times N}$ (L denotes number of lines)?

(c) (**Bonus**[1]) $\overline{\mathbf{Y}}_{\mathcal{T}}$, $\mathbf{A}_{\mathfrak{B}} \in \mathbb{R}^{L \times N}$ and the grid is radial (i.e., without loops)?

Detail your reasoning.

Solution.

(a) **It is possible only for $N \in \{2, 3\}$.**

Given the N diagonal elements, the number of unknowns is $N^2 - N$. Since the matrix is symmetric, finding $\frac{N^2-N}{2}$ non-diagonal elements is sufficient. Knowing the primitive shunt admittance matrix, $\overline{\mathbf{Y}}_{\mathcal{T}}$, we can use N following equations used to express the diagonal elements:

$$\overline{Y}_{kk} = \sum_{i=1, i \neq k}^{N} (-\overline{Y}_{ik}) + \overline{Y}_{\mathcal{T},kk}, \quad k = 1, \ldots, N. \tag{4}$$

To be able to solve the system of N equations (4), the number of unknowns has to be less or equal to the system order:

$$\frac{N^2 - N}{2} \leqslant N \text{ (and } N \geqslant 2) \tag{5}$$

which is valid only for $N \in \{2, 3\}$.

(b) **It is possible only if $N \geqslant L$.**

By knowing the incidence matrix $\mathbf{A}_{\mathfrak{B}}$, we infer that the number of lines is L which imposes L unknown elements of the admittance matrix $\overline{Y}_{ij}$, $i > j$ above the main diagonal. Again, considering the symmetry of the admittance matrix, the number of unknown elements is L. The same system of N equations (4) can be solved if the number of unknowns is less or equal to its order, hence $L \leqslant N$.

(c) (**Bonus**)

Yes, it is always possible.

This yields directly from (b). For a radial network, $N = L + 1$. Therefore, $N = L + 1 \geqslant L$ always holds.

[1] You do not need to answer this question (c) to obtain the maximum score for Problem 3. If you answer correctly, it only compensates for some points you may lose from this Problem 3 (the weight remains 30%).

QUESTION 2 (LOAD FLOW ANALYSIS)

The Load Flow Problem consists of determining the operating conditions of an electrical network. As seen during the lectures, for a network with N nodes, there are $4N$ variables (the active and reactive power injections and the magnitude and angle of the nodal voltages). We can write $2N$ equations, meaning $2N$ variables have to be fixed. These represent the external conditions of the network. The load flow equations link the voltage phasors with the active/reactive power injections. Namely, for $n \in \mathcal{N}$, where $\mathcal{N}$ denotes the set of all the nodes of the network.

$$\bar{S}_n = P_n + jQ_n = \bar{E}_n \underline{I}_n = \bar{E}_n \sum_{h \in \mathcal{N}} \underline{Y}_{nh} \underline{E}_h. \tag{6}$$

In polar coordinates for the nodal voltages and Cartesian for the admittances

$$P_n = E_n \sum_{h \in \mathcal{N}} E_h (G_{nh} \cos(\theta_{nh}) + B_{nh} \sin(\theta_{nh})) \tag{7}$$

$$Q_n = E_n \sum_{h \in \mathcal{N}} E_h (G_{nh} \sin(\theta_{nh}) - B_{nh} \cos(\theta_{nh})) \tag{8}$$

where P_n, Q_n refer to active and reactive powers, E_n, θ_n are the magnitude and phase of the voltage phasor $\bar{E}_n = E_n e^{j\theta_n}$ and $\theta_{nh} = \theta_n - \theta_h$. Typically, a system of nonlinear equations must be solved to determine all the network variables. This can be done by an iterative procedure such as the Newton-Raphson method. Here starting from an initial point $\Delta \mathbf{E}^0, \Delta \boldsymbol{\theta}^0$, at iteration k we compute the corrections in magnitude $\Delta E_n^{(k)}$ and angle $\Delta \theta_n^{(k)}$. Using these corrections, we update the magnitudes and angles until convergence is achieved:

$$\begin{bmatrix} \Delta \mathbf{E} \\ \Delta \boldsymbol{\theta} \end{bmatrix}^{(k)} = \left\{ \begin{bmatrix} \mathbf{J}_{PE} & \mathbf{J}_{P\theta} \\ \mathbf{J}_{QE} & \mathbf{J}_{Q\theta} \end{bmatrix}^{(k)} \right\}^{-1} \times \begin{bmatrix} \Delta \mathbf{P} \\ \Delta \mathbf{Q} \end{bmatrix}^{(k)} \tag{9}$$

$$\begin{bmatrix} \mathbf{E} \\ \boldsymbol{\theta} \end{bmatrix}^{(k+1)} = \begin{bmatrix} \mathbf{E} \\ \boldsymbol{\theta} \end{bmatrix}^{(k)} + \begin{bmatrix} \Delta \mathbf{E} \\ \Delta \boldsymbol{\theta} \end{bmatrix}^{(k)} \tag{10}$$

where $\Delta \mathbf{E}^{(k)}$, $\Delta \boldsymbol{\theta}^{(k)}$, $\Delta \mathbf{P}^{(k)}$, and $\Delta \mathbf{Q}^{(k)}$ are column vectors composed of the elements $\Delta E_n^{(k)}$, $\Delta \theta_n^{(k)}$, $\Delta P_n^{(k)}$, and $\Delta Q_n^{(k)}$, respectively.

Consider now the simple network given in Figure 3. This network is connected to the main grid at node 1. Nodes 2 and 3 represent loads, while a synchronous generator is connected to node 4. The loads are specified as (P_2, Q_2) and (P_3, Q_3), while the synchronous generator delivers power at a fixed voltage (P_4, V_4). The admittance matrix of the network is presented in (11). Finally, the fixed variables in the network are given in Table 1. Consider as base values $A_b = 1$ GW and $V_b = 380$ kV.

Table 1: Externally Specified Injections (Positive means power is injected in the grid).

Case	P_2	Q_2	P_3	Q_3	P_4	V_4
A	−30 MW	−6 MVAr	−50 MW	8 MVAr	10 MW	380 kV
B	−30 MW	6 MVAr	−50 MW	−8 MVAr	10 MW	380 kV

$$\overline{\mathbf{Y}} = \begin{bmatrix} G_{11} + jB_{11} & G_{12} + jB_{12} & 0 + j0 & 0 + j0 \\ G_{21} + jB_{21} & G_{22} + jB_{22} & G_{23} + jB_{23} & G_{24} + jB_{24} \\ 0 + j0 & G_{32} + jB_{32} & G_{33} + jB_{33} & 0 + j0 \\ 0 + j0 & G_{42} + jB_{42} & 0 + j0 & G_{44} + jB_{44} \end{bmatrix} \tag{11}$$

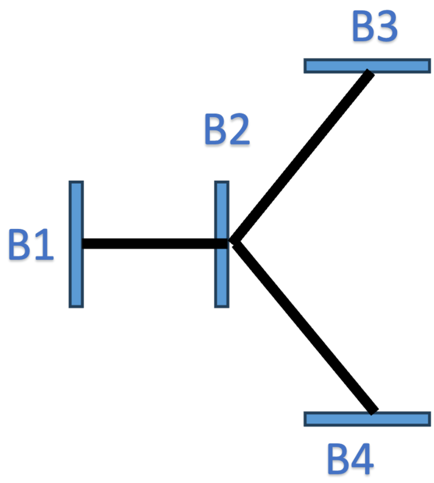


Figure 3: Simple Electric Network

Q2.1 Write the system of equations for solving the load flow problem of the network presented in Fig. 3. The nodal voltages need to be presented in **polar coordinates** and the admittances need to be presented in **Cartesian coordinates**. Be generic by using the elements of the nodal admittance matrix $\overline{\mathbf{Y}} = \mathbf{G} + j\mathbf{B}$ given by (11).

Solution. For each node, we can write the following equations:

- Node 1:
  - $V_1 = 380$ kV $= 1$ p.u.
  - $\theta_1 = 0^\circ$
- Node 2:
  - $P_2 = E_2 \cdot (E_1(G_{21}\cos(\theta_{21}) + B_{21}\sin(\theta_{21})) + E_2G_{22} + E_3 \cdot (G_{23}\cos(\theta_{23}) + B_{23}\sin(\theta_{23})) + E_4 \cdot (G_{24}\cos(\theta_{24}) + B_{24}\sin(\theta_{24})))$
  - $Q_2 = E_2 \cdot (E_1(G_{21}\sin(\theta_{21}) - B_{21}\cos(\theta_{21})) - E_2B_{22} + E_3 \cdot (G_{23}\sin(\theta_{23}) - B_{23}\cos(\theta_{23})) + E_4 \cdot (G_{24}\sin(\theta_{24}) - B_{24}\cos(\theta_{24})))$
- Node 3:
  - $P_3 = E_3 \cdot (E_2(G_{32}\cos(\theta_{32}) + B_{32}\sin(\theta_{32})) + E_3G_{33})$
  - $Q_3 = E_3 \cdot (E_2(G_{32}\sin(\theta_{32}) - B_{32}\cos(\theta_{32})) - E_3B_{33})$
- Node 4:
  - $P_4 = E_4 \cdot (E_2(G_{42}\cos(\theta_{42}) + B_{42}\sin(\theta_{42})) + E_4G_{44})$
  - $V_4 = 380$ kV $= 1$ p.u.

In the Stott approximation, the following assumptions are considered:

- decoupling between the active power variables (i.e. voltage phases) and reactive powers (i.e. voltage modules).
- $B_{il}\cos(\theta_{il}) \approx B_{il}$ since θ_{il} are small, i.e. $\cos(\theta_{il}) \approx 1$
- $G_{il}\sin(\theta_{il}) << B_{il}$ since the values of G_{il} are extremely small
- $Q_i << B_{ii}V_i^2$
- When computing the partial derivatives with respect to E_i, we assume $E_j = 1$ as the voltage variations are small.

Q2.2 Consider the following two Jacobian matrices, written in polar coordinates, which are used to solve the load flow problem in polar coordinates. Which of these belongs to the original load flow equations and which solves the approximated version obtained after the Stott approximation? Explain your reasoning.

$$J_1 = \begin{bmatrix} 1.52 & -0.138 & 12.2 & -3.35 & -4.91 \\ -0.237 & 0.137 & -3.35 & 3.35 & 0 \\ -0.657 & 0 & -4.94 & 0 & 4.94 \\ 12.2 & -3.35 & -1.58 & 0.137 & 0.857 \\ -3.34 & 3.33 & 0.237 & -0.237 & 0 \end{bmatrix} \tag{12}$$

$$J_2 = \begin{bmatrix} 0 & 0 & 12.2 & -3.35 & -4.93 \\ 0 & 0 & -3.35 & 3.35 & 0 \\ 0 & 0 & -4.92 & 0 & 4.92 \\ 12.2 & -3.35 & 0 & 0 & 0 \\ -3.35 & 3.35 & 0 & 0 & 0 \end{bmatrix} \tag{13}$$

Solution. The first Jacobian corresponds to the full load flow. The second is the Stott approximation. In this approximation, the dependence of the voltage angles on the reactive power and of the voltage magnitudes on the active power is neglected, thus making the corresponding terms in the Jacobian equal to zero.

Q2.3 Consider the following figures showing the voltage for this network as computed for the two cases specified in Table 1. Which plot corresponds to which case? Explain your reasoning.

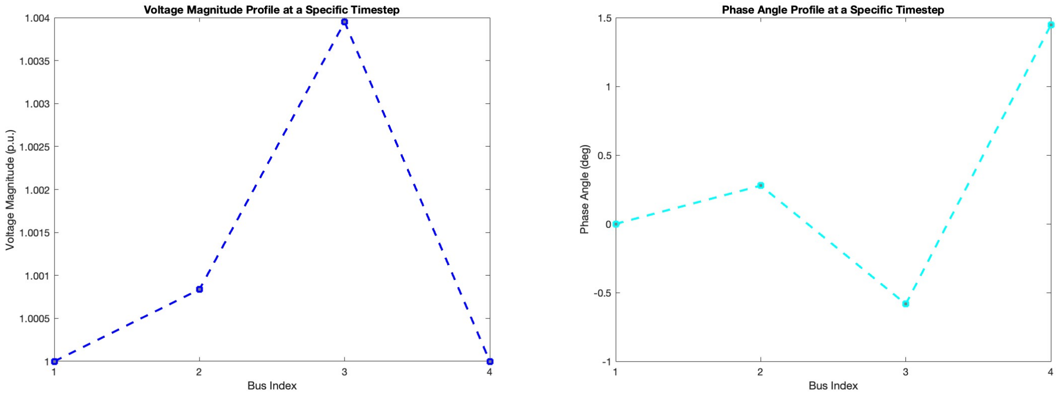


Figure 4: Voltage Magnitudes and Phases Case 1.

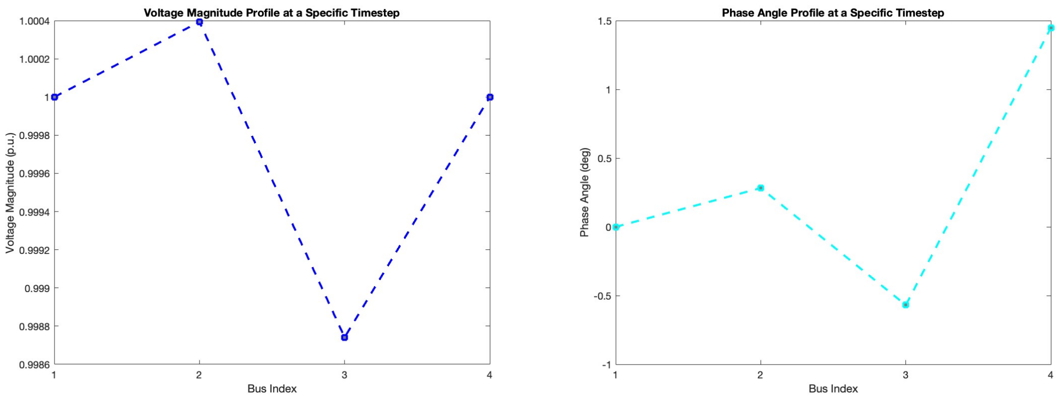


Figure 5: Voltage Magnitudes and Phases Case 2.

Solution. This question can be answered by considering the reactive power injections. Reactive power injections will lead to increased nodal voltages due to the voltage drop occurring between the node injecting and nodes with a fixed reference voltage. Node three, in particular, allows us to see that the first case corresponds to the values from case A, with a significant voltage increase in the first case associated with a high reactive injection, as opposed to a voltage drop in the case of reactive power consumption.

QUESTION 3 (STATE ESTIMATION)

Consider the electric grid shown in Fig. 6, which is described by $\overline{\mathbf{I}} = \overline{\mathbf{Y}} \times \overline{\mathbf{V}}$, where the admittance matrix $\overline{\mathbf{Y}}$ is defined by (14). The slack node has already been removed.

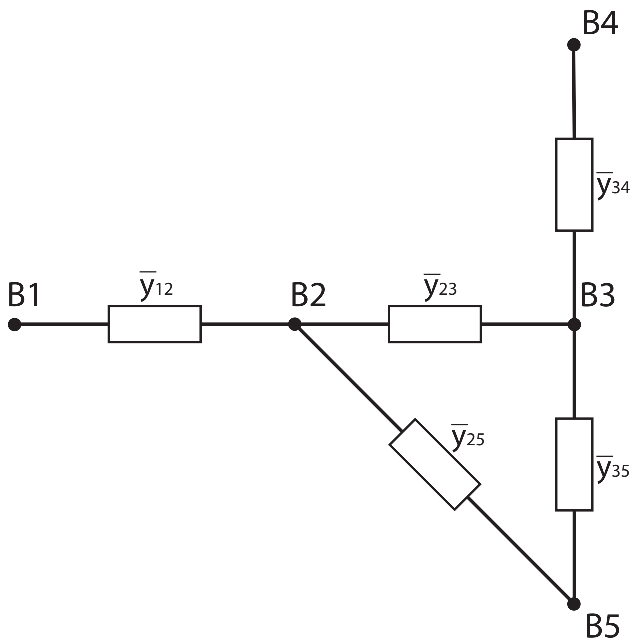


Figure 6: The considered electrical network.

$$\overline{\mathbf{Y}} = \begin{bmatrix} 7.05 - j3.09 & -7.05 + j3.09 & 0.00 & 0.00 & 0.00 \\ -7.05 + j3.09 & 22.75 - j9.35 & -1.61 + j0.07 & 0.00 & -14.09 + j6.19 \\ 0.00 & -1.61 + j0.07 & 14.94 - j2.22 & -6.66 + j1.07 & -6.66 + j1.07 \\ 0.00 & 0.00 & -6.66 + j1.07 & 6.66 - j1.07 & 0.00 \\ 0.00 & -14.09 + j6.19 & -6.66 + j1.07 & 0.00 & 20.76 - j7.2 \end{bmatrix} \text{ p.u.} \tag{14}$$

A linear state estimator needs to be developed for this system. Let $\widetilde{\mathbf{V}}$ and $\widetilde{\mathbf{I}}$ be the vectors of the measured nodal voltage phasors and injected current phasors, respectively. The measurement model at time step t is of the following form:

$$\mathbf{z_t} = \mathbf{H}\mathbf{x_t} + \mathbf{v_t}, \tag{15}$$

where $\mathbf{v_t}$ represents the measurement noise at time step t. The noise is assumed to be white with a normal probability distribution. The measurement vector $\mathbf{z}$ and the state vector $\mathbf{x}$ are formulated in rectangular coordinates. Namely

$$\mathbf{z} = \begin{bmatrix} \mathbf{z}_I \\ \mathbf{z}_V \end{bmatrix}, \text{ with } \mathbf{z}_V = \begin{bmatrix} \Im\{\widetilde{\mathbf{V}}\} \\ \Re\{\widetilde{\mathbf{V}}\} \end{bmatrix}, \ \mathbf{z}_I = \begin{bmatrix} \Im\{\widetilde{\mathbf{I}}\} \\ \Re\{\widetilde{\mathbf{I}}\} \end{bmatrix} \tag{16}$$

$$\mathbf{H} = \begin{bmatrix} \mathbf{H}_I \\ \mathbf{H}_V \end{bmatrix} \tag{17}$$

$$\mathbf{x} = \begin{bmatrix} \Re\{\overline{\mathbf{V}}\} \\ \Im\{\overline{\mathbf{V}}\} \end{bmatrix} \tag{18}$$

Q3.1 Suppose that the slack node has already been removed and the measurement model $\mathbf{H}$ is the following equation (19).

$$\mathbf{H} = \begin{bmatrix} 0 & 0 & 1.07 & -1.07 & 0 & 0 & 0 & -6.66 & 6.66 & 0 \\ 0 & 6.19 & 1.07 & 0 & -7.20 & 0 & -14.09 & -6.66 & 0 & 20.76 \\ 0 & 0.07 & -2.22 & 1.07 & 1.07 & 0 & -1.61 & 14.94 & -6.66 & -6.66 \\ 0 & 0 & -6.66 & 6.66 & 0 & 0 & 0 & -1.07 & 1.07 & 0 \\ 0 & -14.09 & -6.66 & 0 & 20.76 & 0 & -6.19 & -1.07 & 0 & 7.20 \\ 0 & -1.61 & 14.94 & -6.66 & -6.66 & 0 & -0.07 & 2.22 & -1.07 & -1.07 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \text{p.u.} \quad (19)$$

What are the measured quantities and where are they measured? Write down the measurement vector $\mathbf{z}$.

Solution.

We can use the location of the non-zero elements of the $\mathbf{Y} = \mathbf{G} + j\mathbf{B}$ admittance matrix to find the position of the current measurement devices. Since

$$\mathbf{H_I} = \begin{bmatrix} B_{il} & G_{il} \\ G_{il} & -B_{il} \end{bmatrix}, \quad (20)$$

we see that the 1st row of (19) corresponds to the 4th row of (14) and thus to a real part of a current measurement in node $B4$. Following the same analogy, the 2nd and 3th row corresponds to the real current measurement in node $B5$ and $B3$ respectively. The 4th, 5th and 6th row represent the imaginary part of the currents in $B4$, $B5$ and $B3$. Therefore, we can conclude that the current measurement vector is:

$$\mathbf{z_I} = \left[\Im\{\tilde{I}_4\}\ \Im\{\tilde{I}_5\}\ \Im\{\tilde{I}_3\}\ \Re\{\tilde{I}_4\}\ \Re\{\tilde{I}_5\}\ \Re\{\tilde{I}_3\}\right]^\top \quad (21)$$

The location of the real and imaginary voltage measurements can be easily read from the measurement matrix by observing the non-zero elements:

$$\mathbf{z_V} = \left[\Im\{\tilde{V}_1\}\ \Im\{\tilde{V}_3\}\ \Im\{\tilde{V}_5\}\ \Re\{\tilde{V}_1\}\ \Re\{\tilde{V}_3\}\ \Re\{\tilde{V}_5\}\right]^\top \quad (22)$$

The measurement vector becomes:

$$\mathbf{z} = \left[\Im\{\tilde{I}_4\}\ \Im\{\tilde{I}_5\}\ \Im\{\tilde{I}_3\}\ \Re\{\tilde{I}_4\}\ \Re\{\tilde{I}_5\}\ \Re\{\tilde{I}_3\}\ \Im\{\tilde{V}_1\}\ \Im\{\tilde{V}_3\}\ \Im\{\tilde{V}_5\}\ \Re\{\tilde{V}_1\}\ \Re\{\tilde{V}_3\}\ \Re\{\tilde{V}_5\}\right]^\top \quad (23)$$

Q3.2 Does the present placement of measurements guarantee that the network is observable? Explain why. If the network is not observable, propose an extended set of PMU measurements that satisfies the necessary observability criteria.

Solution. The system is observable if $\mathbf{H}$ is full column rank (i.e. $rank(\mathbf{H}) = \#_{col} = 10$). The matrix $\mathbf{H}$ has 12 rows and the columns are linearly independent. Therefore, the present measurement placement **guarantees observability**.

SCIPER: ____________________

First name: ________________________ Family name: ________________________

EXAM SMART GRID TECHNOLOGIES 2023 (Total Duration: 3 hours)
With Solutions

PROBLEM 4 (WEIGHT: 28%)

QUESTION 1

Consider the grid in Figure 1. Bus 1 is the slack bus (V, θ bus). There is one generator at bus 1 and one at bus 2, with active power injections $g_1 \geq 0, g_2 \geq 0$. The maximum possible value of g_1 [resp. g_2] is g_1^{MAX} [resp. g_2^{MAX}]. There is one load at bus 3, with active power consumption $\ell_3 \geq 0$. The maximum generator active powers, the generation costs, the maximum branch power flows and the line admittances are indicated in the figure. All values are in per unit. The shunt admittances $b_{i,0}$ (admittances between bus and ground) are assumed to be equal to 0.

1. We assume in this item that the value of $\ell_3 = 0.12$ p.u. is known. We want to set the generation active powers g_1, g_2 in order to minimize the total cost of generation.

   (a) Determine the optimal values of g_1, g_2 obtained with the DC-approximation. Note: we don't need to compute the voltage angles now.

   Solution. With the DC approximation there is no active power loss, so the power balance equations give

$$g_1 + g_2 = \ell_3$$

   the branch power constraints are much larger than the total maximum generation of both $G1$ and $G2$ so they are never constraining. Therefore, we obtain the optimization problem:

$$\begin{aligned} \min 2g_1 + g_2 \quad & \text{over } g_1, g_2 \\ \text{s.c.} \quad & g_1 + g_2 = 0.12 \\ & 0 \leq g_1 \leq 0.3 \\ & 0 \leq g_2 \leq 0.1 \end{aligned}$$

   We can eliminate one of the variables because of the equality constraint. Let us eliminate g_1 by using $g_1 = 0.12 - g_2$. We obtain the equivalent problem:

$$\begin{aligned} \min\,(2(0.12 - g_2) + g_2) \quad & \text{over } g_2 \\ \text{s.c.} \quad & 0 \leq 0.12 - g_2 \leq 0.3 \\ & 0 \leq g_2 \leq 0.1 \end{aligned}$$

   which is equivalent to

$$\begin{aligned} \min(0.24 - g_2) \quad & \text{over } g_2 \\ \text{s.c.} \quad & -0.18 \leq g_2 \leq 0.12 \\ & 0 \leq g_2 \leq 0.1 \end{aligned}$$

which is equivalent to

$$\min(0.24 - g_2) \quad \text{over } g_2$$
$$\text{s.c.} \quad 0 \leq g_2 \leq 0.1$$

and the optimum is for $g_2 = 0.1$, hence $g_1 = 0.02$.

(b) Let P_{ij} be the active branch power flowing from bus i to bus j, seen at bus i, predicted by the DC approximation at this optimal solution. Also, let θ_i be the voltage angle at bus i. Compute the voltage angles, in mrad, predicted by the DC approximation (recall that g_1, g_2 have the values obtained in the previous question). Say in which direction active power flows on the line $1-2$, according to the DC approximation.

Solution.

The voltage angles are $\theta_1 = 0, \theta_2, \theta_3$. We have $B_{12} = B_{13} = 1$ and $B_{23} = 2$, thus the DC load flow equations give

$$\begin{aligned} P_{12} &= -\theta_2 \\ P_{13} &= -\theta_3 \\ P_{23} &= 2(\theta_2 - \theta_3) \end{aligned}$$

The DC power balance equations at all buses give

$$\begin{aligned} P_{12} + P_{13} &= g_1 = 0.02 \\ P_{21} + P_{23} &= g_2 = 0.1 \\ P_{32} + P_{31} &= \ell_3 = -g_1 - g_2 \end{aligned}$$

Note that the last equation above is redundant and we ignore it in the rest. Replacing P_{ij} by their expressions in function of the angles in the previous equations give:

$$\begin{aligned} -\theta_2 - \theta_3 &= 0.02 \\ 3\theta_2 - 2\theta_3 &= 0.1 \end{aligned}$$

This is a system of two equations with two unknowns, which we can solve and obtain:

$$\begin{aligned} \theta_2 &= 0.012 \\ \theta_3 &= -0.032 \end{aligned}$$

The above values are in radians, i.e., $\theta_2 = 12\text{mrad}$ et $\theta_3 = -32\text{mrad}$. The complex voltages predicted by the DC approximation are

$$v_1 = 1, \; v_2 = 1 + 0.012j, \; v_3 = 1 - 0.032j$$

A correct answer is also

$$v_1 = 1, \; v_2 = e^{0.012j}, \; v_3 = e^{-0.032j}$$

since the DC approximation relies on the approximation $e^{j\theta} \approx 1 + j\theta$. Recall that the DC approximation assumes that voltage magnitudes are equal to 1.

Since $\theta_2 > 0$, we have $P_{12} < 0$, i.e., the active power flows from bus 2 to bus 1 on the line $1-2$.

(c) Since the optimal values in the previous question were found using an approximation, the real values of power flows will be different. In this item we assume that ℓ_3 has the same value as in the previous question, and we set g_2 to the optimal value found earlier. Since bus 1 is the slack bus, it ensures the balance of active and reactive power. It follows that the true value of active power generated by $G1$ will not be equal to the value of g_1 found in the previous question but will be equal to the value required by power balance. Let g_1^* be this value. Also let P_{ij}^* be the resulting true value of the active power flowing on the branch from bus i to bus j, seen at bus i.

Say what is true in the table below. There is exactly one correct answer per column. Give your answer in the table by putting ⊠ in the correct box.

The usual rules of quizzes apply: (i) if you cross the correct box in one column, you obtain the full score for this column; (ii) if you cross zero or more than one box in one column, your score is 0 for this column; (iii) if you cross exactly one box in one column and it is incorrect your score for this column is negative, namely a penalty equal to half the score.

☒	$\|P_{23}+P_{32}\| < \|P_{23}^*+P_{32}^*\|$	☐	$P_{32} < P_{32}^*$	☒	$g_1 < g_1^*$
☐	$\|P_{23}+P_{32}\| = \|P_{23}^*+P_{32}^*\|$	☐	$P_{32} = P_{32}^*$	☐	$g_1 = g_1^*$
☐	$\|P_{23}+P_{32}\| > \|P_{23}^*+P_{32}^*\|$	☐	$P_{32} > P_{32}^*$	☐	$g_1 > g_1^*$

Solution. $P_{23}+P_{32}$ is the active power loss in the line 2–3, which is 0 for the DC approximation and is non zero for the true value $P_{23}^*+P_{32}^*$.

There was a bug in the second column. One of the three answers is correct, but obtaining the correct answer requires solving a load-flow problem. As the complexity of solving this question is higher than expected, the second column was not graded.

$g_1^*+g_2=\ell_3+$true power loss on all lines whereas $g_1+g_2=\ell_3$ hence $g_1^*>g_1$.

2. We now assume that the load active power consumption ℓ_3 is uncertain, but we have a forecast ℓ_3^0 with some prediction uncertainty Δ, i.e. we assume that

$$\ell_3^0-\Delta \quad\le\quad \ell_3 \le \ell_3^0+\Delta$$

where $0\le\Delta\le\ell_3^0$.

We want to fix in advance a value of g_2 by minimizing the expected cost of generation, which is the cost obtained when we assume that the load ℓ_3 is equal to ℓ_3^0. In this question, we use the DC approximation. Write a linear programming problem to solve this problem. Your linear programming problem should not contain the quantifier $\forall$ in the constraints.

Numerical application: for the values shown on Figure 1, when $\ell_3^0=0.12$ p.u., for which values of Δ is the problem feasible ?

Solution. The expected cost is $c_1g_1+c_2g_2$ computed when we assume that $\ell_3=\ell_3^0$, i.e. $g_1=-g_2+\ell_3^0$. Thus the expected cost is

$$-c_1g_2+c_1\ell_3^0+c_2g_2=(c_2-c_1)g_2+c_1\ell_3^0=2\ell_3^0-g_2 \tag{1}$$

As before, given the values of g_1^{MAX} and g_2^{MAX}, we can ignore constraints on branch power flows, i.e. the constraints are

$$0\le g_1\le g_1^{\text{MAX}}$$
$$0\le g_2\le g_2^{\text{MAX}}$$

If we choose g_2 then

$$g_1=-g_2+\ell_3$$

and the constraints become

$$0 \leq -g_2 + \ell_3 \leq g_1^{\mathrm{MAX}}, \quad \forall \ell_3 \text{ s.t. } \ell_3^0 - \Delta \leq \ell_3 \leq \ell_3^0 + \Delta \tag{2}$$
$$0 \leq g_2 \leq g_2^{\mathrm{MAX}} \tag{3}$$

(2) is equivalent to

$$g_2 \leq \ell_3, \forall \ell_3 \text{ s.t. } \ell_3^0 - \Delta \leq \ell_3 \leq \ell_3^0 + \Delta \tag{4}$$

and

$$g_2 \geq \ell_3 - g_1^{\mathrm{MAX}}, \forall \ell_3 \text{ s.t. } \ell_3^0 - \Delta \leq \ell_3 \leq \ell_3^0 + \Delta \tag{5}$$

(4) is equivalent to

$$g_2 \leq \min \ell_3 \text{ over } \ell_3 \text{ s.t. } \ell_3^0 - \Delta \leq \ell_3 \leq \ell_3^0 + \Delta \tag{6}$$
$$g_2 \leq \ell_3^0 - \Delta \tag{7}$$

(5) is equivalent to

$$g_2 \geq \max\left(\ell_3 - g_1^{\mathrm{MAX}}\right) \text{ over } \ell_3 \text{ s.t. } \ell_3^0 - \Delta \leq \ell_3 \leq \ell_3^0 + \Delta \tag{8}$$
$$g_2 \geq \ell_3^0 + \Delta - g_1^{\mathrm{MAX}} \tag{9}$$

thus (2) is equivalent to

$$\ell_3^0 + \Delta - g_1^{\mathrm{MAX}} \leq g_2 \leq \ell_3^0 - \Delta$$

Putting things together, we obtain the optimization problem:

$$\min\left(2\ell_3^0 - g_2\right) \text{ over } g_2 \text{ s. c. } \ell_3^0 + \Delta - g_1^{\mathrm{MAX}} \leq g_2 \leq \ell_3^0 - \Delta$$
$$0 \leq g_2 \leq g_2^{\mathrm{MAX}}$$

Numerical Application: the problem becomes

$$\min\left(2\ell_3^0 - g_2\right) \text{ over } g_2 \text{ s. c.}$$
$$-0.18 + \Delta \leq g_2 \leq 0.12 - \Delta$$
$$0 \leq g_2 \leq 0.1$$

i.e. the constraints are

$$g_2 \geq -0.18 + \Delta$$
$$g_2 \geq 0$$
$$g_2 \leq 0.12 - \Delta$$
$$g_2 \leq 0.1 \qquad (d)$$

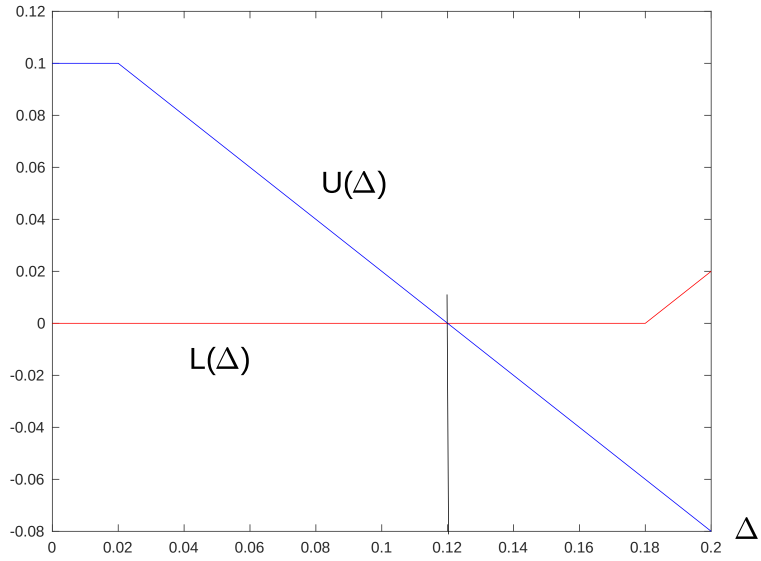
which is equivalent to

$$g_2 \geq \max(-0.18 + \Delta, 0)$$
$$g_2 \leq \min(0.12 - \Delta, 0.1)$$

The problem is feasible if and only if

$$L(\Delta) \overset{\text{def}}{=} \max(-0.18 + \Delta, 0) \leq \min(0.12 - \Delta, 0.1) \overset{\text{def}}{=} U(\Delta)$$

We plot $L(\Delta)$ and $U(\Delta)$ and see that $L(\Delta) \leq U(\Delta)$ if and only if $\Delta \leq 0.12$.



In summary, the problem is feasible if and only if $0 \leq \Delta \leq 0.12$.

QUESTION 3

The gridSteer system manages a home system made of a boiler with a heat pump, a photovoltaic (PV) roof, and a set of other loads. The connection to the main grid can be used either to import or to export energy. The cost of imported energy and the payment for exported energy are time-dependent but are known in advance. The goal of the gridSteer system is to decide, at every discrete time t, which amount of energy should be bought from / sold to the main grid and which amount should be used to heat water in the boiler.

Notation:

- $u_t \geq 0$: energy spent to increase the temperature of the boiler during time slot t; we must have $u^{\min} \leq u_t \leq u^{\max}$ where $u^{\min}$ and $u^{\max}$ are positive constants.
- T_t: internal temperature of the boiler;
- $d_t \geq 0$: heat drawn from the boiler by the home occupants;
- v_t: energy drawn from the main grid; v_t can be ≥ 0 (net consumption) or ≤ 0 (net production); we must have $v^{\min} \leq v_t \leq v^{\max}$ where $v^{\min} < 0 < v^{\max}$ are constants.
- $\ell_t \geq 0$: energy consumed by the other loads;
- $g_t \geq 0$: energy produced by the photovoltaic roof.

We ignore grid losses, so that

$$v_t + g_t = u_t + \ell_t$$

The temperature of the boiler is given by the following dynamic model

$$T_{t+1} = T_t + \eta u_t - d_t$$

where η is the coefficient of efficiency of the combined heat pump / boiler system.

The boiler temperature should ideally lie in the range $[T^{\min}, T^{\max}]$. To reach this goal, gridSteer uses the penalty function

$$\varphi(T) = \begin{cases} -T + T^{\min} & \text{if } T \leq T^{\min} \\ 0 & \text{if } T^{\min} \leq T \leq T^{\max} \\ T - T^{\max} & \text{if } T \geq T^{\max} \end{cases}$$

The price of imported energy is α_t and the revenue per unit of exported energy is β_t, with $0 < \beta < \alpha$ (because of grid taxes). Thus, when the home exports energy to the main grid ($v_t < 0$), it receives a payment of $-\beta_t v_t$; when it consumes energy from the main grid ($v_t > 0$), it pays $\alpha_t v_t$.

1. Write the function $\varphi()$ as a maximum of linear functions.

   Solution. By inspection we find that

   $$\varphi(T) = \max\left(-T + T^{\min}, 0, T - T^{\max}\right)$$

2. The cost of energy exchange with the main grid is:

   $$c_t(v) = \begin{cases} \beta_t v & \text{if } v \leq 0 \text{ (energy is exported, cost is negative)} \\ \alpha_t v & \text{if } v \geq 0 \text{ (energy is imported, cost is positive)} \end{cases} \tag{19}$$

   For every fixed value of t, write the function $c_t()$ as a maximum of linear functions.

   Solution. We can verify that

   $$c_t(v) = \max\left(\beta_t v, \alpha_t v\right) \tag{20}$$

   Indeed, $\alpha_t > \beta_t > 0$ hence:

   - if $v \leq 0$, $\alpha_t v \leq \beta_t v$ and the right-handside of (20) is $\beta_t v$, i.e. is equal to $c_t(v)$ as given by (19);
   - if $v \geq 0$, $\alpha_t v \geq \beta_t v$ and the right-handside of (20) is $\alpha_t v$, i.e. is equal to $c_t(v)$ as given by (19).

3. gridSteer uses an MPC with horizon equal to H time steps in order to set the value of u_t, starting from a known value T_t of the state of the boiler. For this MPC, we assume that gridSteer has forecasts $\hat{\ell}_{t:t+H-1}$, $\hat{g}_{t:t+H-1}$ and $\hat{d}_{t:t+H-1}$ of the load, the PV generations and the extraction of heat from the boiler. The cost function minimized by the MPC at time step s is $\varphi(T_{s+1}) + c_s(v_s)$. Write a *linear* formulation of this MPC.

   Solution. The MPC assumes that T_t is known and solves the optimization problem

   $$\min \sum_{s=t}^{t+H-1} \left(\varphi(T_{s+1}) + c_t(v_s)\right)$$

   over $u_{t:t+H-1}, v_{t:t+H-1}, T_{t+1,t+H}$

   s. c. $v_s + \hat{g}_s = u_s + \hat{\ell}_s$ for $s = t : t + H - 1$

   $T_{s+1} = T_s + \eta u_s - \hat{d}_s$ for $s = t : t + H - 1$

   $u^{\min} \leq u_s \leq u^{\max}$ for $s = t : t + H - 1$

   $v^{\min} \leq v_s \leq v^{\max}$ for $s = t : t + H - 1$

Note that in this optimization problem the initial state T_t is given. Then gridSteer implements u_t and v_t.

This is not a linear formulation because of the functions $\varphi()$ and $c_t()$. But using the results from the previous two questions and the max-removal transformation, we obtain the following linear MPC:

$$
\begin{aligned}
&\min \sum_{s=t}^{t+H-1} (\varepsilon_{s+1} + \xi_s) \\
&\text{over } u_{t:t+H-1}, v_{t:t+H-1}, T_{t+1,t+H}, \varepsilon_{t+1:t+H}, \xi_{t:t+H-1} \\
&\text{s. c. } \varepsilon_s \geq -T_s + T^{\min} \text{ for } s = t+1 : t+H \\
&\qquad \varepsilon_s \geq 0 \text{ for } s = t+1 : t+H \\
&\qquad \varepsilon_s \geq T_s - T^{\max} \text{ for } s = t+1 : t+H \\
&\qquad \xi_s \geq \alpha_s v_s \text{ for } s = t : t+H-1 \\
&\qquad \xi_s \geq \beta_s v_s \text{ for } s = t : t+H-1 \\
&\qquad v_s + \hat{g}_s = u_s + \hat{\ell}_s \text{ for } s = t : t+H-1 \\
&\qquad T_{s+1} = T_s + \eta u_s - \hat{d}_s \text{ for } s = t : t+H-1 \\
&\qquad u^{\min} \leq u_s \leq u^{\max} \text{ for } s = t : t+H-1 \\
&\qquad v^{\min} \leq v_s \leq v^{\max} \text{ for } s = t : t+H-1
\end{aligned}
$$

SMART GRID TECHNOLOGIES EXAM - PB4
FIGURES

For your convenience, you can separate this sheet from the main document. Do not write your solution on this sheet, use only the main document. Do not return this sheet.

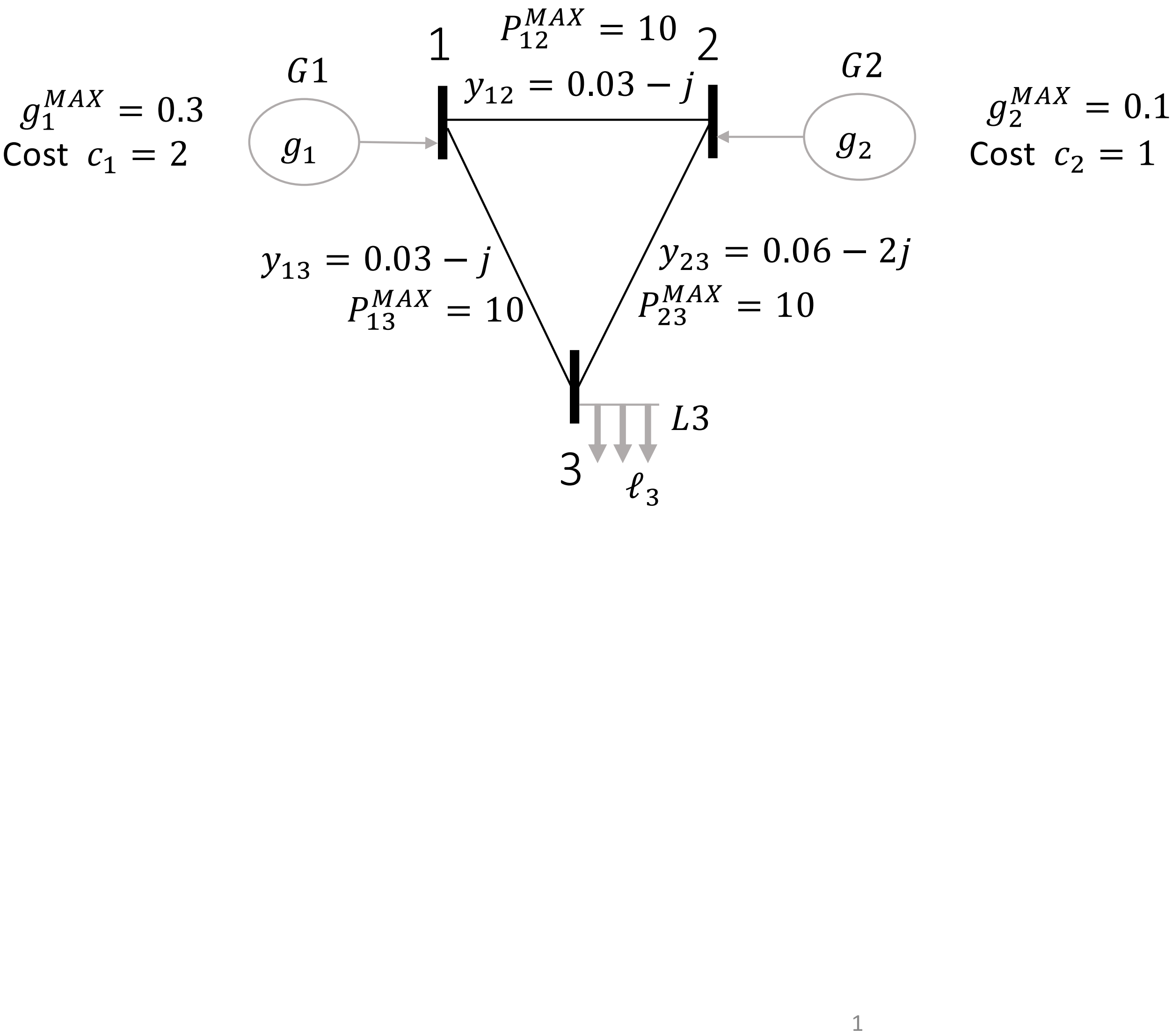


Figure 1: The grid used in Question 1.